

SHELLABLE NONPURE COMPLEXES AND POSETS. I

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ABSTRACT. The concept of shellability of complexes is generalized by deleting the requirement of purity (i.e., that all maximal faces have the same dimension). The usefulness of this level of generality was suggested by certain examples coming from the theory of subspace arrangements. We develop several of the basic properties of the concept of nonpure shellability.

Doubly indexed f -vectors and h -vectors are introduced, and the latter are shown to be nonnegative in the shellable case. Shellable complexes have the homotopy type of a wedge of spheres of various dimensions, and their Stanley-Reisner rings admit a combinatorially induced direct sum decomposition.

The technique of lexicographic shellability for posets is similarly extended from pure posets (all maximal chains of the same length) to the general case. Several examples of nonpure lexicographically shellable posets are given, such as the k -equal partition lattice (the intersection lattice of the k -equal subspace arrangement) and the Tamari lattices of binary trees. This leads to simplified computation of Betti numbers for the k -equal arrangement. It also determines the homotopy type of intervals in a Tamari lattice and in the lattice of number partitions ordered by dominance, thus strengthening some known Möbius function formulas.

The extension to regular CW complexes is briefly discussed and shown to be related to the concept of lexicographic shellability.

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